Enrollment No: ____________________ Exam Seat No: ____________________

C.U.SHAH UNIVERSITY

Summer Examination-2022

Subject Name: Real Analysis - II

Subject Code: 4SC06REA1 **Branch: B.Sc. (Mathematics)**

Semester: 6 **Date: 02/05/2022** **Time: 02:30 To 05:30** **Marks: 70**

Instructions:

(1) Use of Programmable calculator & any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

Q-1 Attempt the following questions: **[14]**

a) State Darboux's theorem for Riemann integration. (02)

b) Let $I = [1,6]$ and $P = \{1,3,5,6\}$ be the partition of $[1,6]$, Define $f: [1,6] \to \boldsymbol{R}$ by $f(x) = x^2$ then find $L(P, f)$. (02)

c) Show that the function $f: R \to R$ defined by $f(x) = \begin{cases} x & 0 \le x < 1 \\ 1 & x \ge 1 \end{cases}$ is not derivable at $x = 1$. (02)

d) Evaluate : $\lim_{x\to 0} \frac{\log x}{\cot x}$. (02)

e) State Cauchy's Mean Value Theorem. (02)

f) State Fundamental Theorem of Calculus. (01)

g) Define :Exponential Function. (01)

h) True/False : The radius of convergence of the series $\sum_{n=0}^{\infty}(n+1)x^n$ is 1. (01)

i) Define: Norm of Partition. (01)

Attempt any four questions from Q-2 to Q-8.

Q-2 Attempt all questions **[14]**

a) If a bounded function f is integrable on $[a, b]$ then show that it is also integrable on $[a, c]$ and $[c, b]$ where $c \in (a, b)$also justify its converse. (07)

b) Prove that every constant function is R-integrable on any closed interval ,also find the value of Riemann integration. (05)

c) If P^* is a refinement of a partition P then show that $U(P^*, f) \le U(P, f)$. (02)

Q-3 Attempt all questions **[14]**

a) State and prove necessary and sufficient condition for a bounded function to be Riemann integrable on closed interval. (07)

b) If $f_1, f_2 \in R[a,b]$ then prove that $f_1 f_2 \in R[a,b]$.Can we say that $f \in R[a,b]$ if $f^2 \in R[a,b]$?Justify. (07)

Q-4 Attempt all questions **[14]**

a) Prove :$\frac{\pi^2}{6} \leq \int_0^\pi \frac{x}{2+\cos x} \leq \frac{\pi^2}{2}$.Also state the result you use. (06)

b) Show that $\tan^{-1} x = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \cdots, -1 \leq x \leq 1$. (04)

c) Show that$\frac{v-u}{1+v^2} < \tan^{-1} v - \tan^{-1} u < \frac{v-u}{1+u^2}$if $0 < u < v$.Also deduce that (04)

$$\frac{\pi}{4} + \frac{3}{25} < \tan^{-1}\frac{4}{3} < \frac{\pi}{4} + \frac{1}{6}.$$

Q-5 Attempt all questions **[14]**

a) State and prove Lagrange's mean-value theorem. (05)

b) Show that $x - \frac{x^2}{2} < \log(1+x) < x - \frac{x^2}{2(1+x)}$ for $\forall\, x > 0$. (05)

c) Verify Rolle's theorem for the function $f(x) = (x-a)^m (x-b)^n$ where m and n are positive integerson$[a,b]$. (04)

Q-6 Attempt all questions **[14]**

a) Let $\{f_n\}$ be a sequence of differentiable functions on $[a,b]$ such that it converges at least one point $x_0 \in [a,b]$.If the sequence of differentials $\{f_n'\}$ converges uniformly to G on $[a,b]$ then show that given sequence$\{f_n\}$ converges uniformly on $[a,b]$ to f and $f'(x) = G(x)$for all $x \in [a,b]$. (09)

b) Prove that a every derivable function is continuous .Does the converse true?Justify. (05)

Q-7 Attempt all questions **[14]**

a) Test the uniform convergence of the sequence $\{f_n\}$ on $[0,100]$,where $f_n(x) = \frac{nx}{1+n^2x^2}$. (05)

b) Show that if the power series $\sum a_n x^n$ converges for $x = x_0$ then it is absolutely convergent for every $x = x_1$,when $|x_1| < |x_0|$. (05)

c) Evaluate :$\lim_{x\to 0} \frac{xe^x - \log(1+x)}{x^2}$. (04)

Q-8 Attempt all questions **[14]**

a) State and prove Able's Theorem (second form). (10)

b) Find the radius of convergence of the series $\frac{1}{2}x + \frac{1\cdot 3}{2\cdot 5}x^2 + \frac{1\cdot 3\cdot 5}{2\cdot 5\cdot 8}x^3 + \cdots$ (04)